DOCUMENT RESUME

ED 402 340 TM 025 927

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TITLE How Do Extreme Schools Change the Interpretation of Results in School Effectiveness Research? Effects of Outlying Second-Level Variables in HLM.
PUB DATE Apr 96
NOTE 12p.; Paper presented at the Annual Meeting of the American Educational Research Association (New York, NY, April 8-12, 1996).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Effective Schools Research; Elementary Secondary Education; *Error of Measurement; *Estimation (Mathematics); *School Effectiveness; Simulation; *Statistical Analysis
IDENTIFIERS Fixed Effects; *Hierarchical Linear Modeling; Multilevel Analysis; *Outliers

ABSTRACT

This study examined the robustness of the estimation of fixed-effects in multilevel analysis, as might occur in the conduct of school-effects studies with outlying schools. Outlying values for both intercepts and slopes for individual schools were modeled separately to determine the effects of extreme values of second-level variables on the fixed-effect parameter estimation. A total of seven data sets were generated for the simulation. Under the conditions investigated in this study, adding a cluster of outlying schools had little effect on the estimation of gamma 10 and gamma 11. However, the standard error of gamma 01 increased, thereby increasing the conservativeness of the test of significance of gamma 01. This occurred for clusters of outlying slopes, intercepts, or a combination of both. Introducing a single outlying school also increased the standard error of gamma 01, the effects being more dramatic when the outlier was an extreme slope. (Author/SLD)

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ED 402 340

How do Extreme Schools Change the Interpretation of Results in School-Effectiveness Research?: Effects of Outlying Second-Level Variables in HLM.

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This paper was presented at the Annual Meeting of the American Educational Research Association, New York, NY, April, 1996

TM025927

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Abstract

This study examined the robustness of the estimation of fixed-effects in multilevel analysis, as might occur when conducting school-effects studies with outlying schools. Outlying values for both intercepts and slopes for individual schools were modeled separately to determine the effects of extreme values of second-level variables on the fixed-effect parameter estimation. Under the conditions investigated in this study, adding a cluster of outlying schools had little effect on the estimation of γ_{10} and γ_{11}, however the standard error of γ_{01} increased, thereby increasing the conservativeness of the test of significance of γ_{01}. This occurred for clusters of outlying slopes, intercepts, or a combination of both. Introducing a single outlying school also increased the standard error of γ_{01}, the effects being more dramatic when the outlier was an extreme slope.

How do Extreme Schools Change the Interpretation of Results in School-Effectiveness Research?: Effects of Outlying Second-Level Variables in HLM.

Theoretical Perspective

Multilevel analysis has recently replaced multiple linear regression,(MLR) as the method of choice for school-effects research (Mendro, Webster, Bembry, & Orsak, 1995; Webster, Mendro, & Almaguer, 1993). Bryk and Raudenbush, (1992) give at least two explanations for the appropriateness of multilevel analysis for school-effects research. First, random variation and structural effects may exist at more than one level, and therefore a correctly specified model is a multilevel model in which fixed and random effects can be estimated at each level. Second, the assumption of independence of errors in MLR is violated when there is intraclass correlation, as one would find in such hierarchically nested data.

The models for a simple multilevel analysis with one level-1 and one level-2 fixed-effect variable would be:

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1j} + r_{ij} \quad (1)$$

for the level-1 model and

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j} \quad (2)$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}W_j + u_{1j} \quad (3)$$

for the level-2 models. The level-1 model is analogous to a simple regression model, where r_{ij} is the level-1 error. The parameters from level-1 become outcome variables in the level-2 models, which are predicted by second-level effects. u_{0j} and u_{1j} are the random error components of level 2. Upon inspection of these

models, it is apparent that aberrant or extreme values for β_{0j} and β_{1j} will affect the estimation of parameters in the second-level models. These discrepant schools could have an undue influence on the estimation of the second-level fixed effects of the model. Therefore, Bryk and Raudenbush, (1992) suggested that cross-level exploratory analyses be performed when conducting school-effects studies to determine if there are schools with outlying intercepts or slopes.

The effects of outliers have been extensively studied in ordinary least squares regression analysis. Rousseeuw and van Zomeren (1990) identify three types of outliers: a) vertical outliers in which there is a large residual, b) leverage point outliers, which are consistent with the relationships found in the rest of the data and c) leverage point outliers, which significantly alter the relationships by their inclusion, as well as having high leverage. Outliers exert effects in not only the results of the main and interaction-effects significance tests from GLM, but also in the estimation of the model parameters (Douzenis & Rakow, 1987; Hecht, 1991).

This study is designed to examine the robustness of the estimation of fixed-effects in multilevel analysis, when conducting school-effects studies with outlying schools. Outlying values for both intercepts and slopes for individual schools are modeled separately to determine the effects of extreme values of second-level variables on the fixed-effect parameter estimation.

Data Source

A total of seven data sets were generated for this study. Data set I was simulated to serve as the major body of data. This dataset consisted of 250 schools with 50 students in each school. Three datasets were generated based on the original dataset, but containing 10%: a) schools with extreme intercepts, b) schools with extreme slopes, or c) schools with extreme intercepts and slopes. The extreme intercept is analogous to adding vertical outliers. These were generated by increasing the mean approximately 3 standard deviations units

higher. The slope outliers were generated by reducing the correlation between X and Y from .7 to .4, holding s_x and s_y constant. The dataset size was maintained at 250 level II observations or schools. The three new datasets were termed Outlier-I, Outlier-S, and Outlier-SI, respectively. Multiple regression diagnostics tests of the schools, were examined to determine the most extreme outliers for each of the three datasets, as well as to determine that each dataset contained 10% outliers. Specifically, the intercept and slope DF Betas were checked to ensure that they increased for the outlying observations.

To produce the last three data sets, extreme outlying observations were substituted for observations in the original dataset. One school was removed from the original data set, and the most extreme outlying school found in the analysis of Outlier-I was inserted in its place. This new dataset was termed Single-I. This process was repeated for Outlier-S, and Outlier-SI, producing the new datasets Single-S, and Single-SI, respectively.

Procedures and Results

A two-level hierarchical linear modeling program (HLM) was used to conduct the multilevel analysis for this study. Separate HLM analyses were conducted for each dataset. Table 1 presents the descriptive statistics of the seven datasets. It can be seen that adding the 10% outlying schools increased the standard deviations of the original dataset. However, adding the one outlying school had little effect on the standard deviations.

Table 1

Descriptive Statistics

	Outlier	Variable	N	Mean	SD	Minimum	Maximum
No Outlier		X	12,500	180.04	9.04	142.51	214.02
		Y	12,500	192.03	13.02	140.30	241.76
		W	250	5.60	0.14	5.21	5.99
10% Outliers	Intercept	X	12,500	180.04	9.04	142.51	214.02
		Y	12,500	197.03	19.34	140.30	282.38
		W	250	5.60	0.14	5.21	5.99
	Slope	X	12,500	180.02	9.06	142.51	214.02
		Y	12,500	184.20	26.89	73.44	241.76
		W	250	5.60	0.14	5.21	5.99
	Intercept & Slope	X	12,500	180.05	9.07	142.51	214.02
		Y	12,500	188.27	17.25	106.95	241.76
		W	250	5.60	0.14	5.21	5.99
Single Outlier	Intercept	X	12,500	180.04	9.04	142.51	214.02
		Y	12,500	192.23	13.33	140.30	276.10
		W	250	5.60	0.14	5.21	5.99
	Slope	X	12,500	180.05	9.05	142.51	214.02
		Y	12,500	191.73	13.87	94.20	241.76
		W	250	5.60	0.14	5.21	5.99
	Intercept & Slope	X	12,500	180.05	9.04	142.51	214.02
		Y	12,500	191.89	13.22	127.53	241.76
		W	250	5.60	0.14	5.21	5.99

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Table 2

Average Regression Coefficient and Reliability Estimation for Level I

	Outlier Type	β_0	Reliability$_{\beta 0}$	β_1	Reliability$_{\beta 1}$
No Outlier		192.00	0.486	1.01	0.043
	Intercept	197.00	0.992	1.01	0.046
10%	Slope	184.20	0.997	0.97	0.465
Outliers	Intercept & Slope	188.30	0.986	0.97	0.432
	Intercept	192.20	0.853	1.01	0.053
Single	Slope	191.70	0.936	1.01	0.059
Outlier	Intercept & Slope	191.90	0.803	1.01	0.080

Table 2 presents the level I regression coefficients and the corresponding reliability estimates.[1] It can be seen that the reliability estimates for all the datasets with outliers were increased. The increase in the school differences in the parameter estimates increased ι_{qq} which increased the reliability estimates. The average parameter estimate for β_0 changed when outliers were added. This occurred for both 10% outliers and single outliers. The average coefficient estimate for β_1 did not change when intercept outliers were added. However, it did change the average estimate of β_1 when 10% slope or slope and intercept outliers were added.

Table 3

Regression Coefficient for Level II

	Outlier Type	γ_{00}	SE γ_{00}	γ_{01}	SE γ_{01}	γ_{10}	SE γ_{10}	γ_{11}	SE γ_{11}
No Outlier		192.0*	0.116	4.51*	0.804	1.01*	0.009	0.07	0.066
	Intercept	197.0*	0.911	-10.7	6.333	1.01*	0.010	0.07	0.066
10%	Slope	184.2*	1.498	1.06	10.412	0.97*	0.013	0.05	0.091
Outliers	Intercept & Slope	188.3*	0.726	2.54	5.048	0.97*	0.013	0.08	0.089
	Intercept	192.2*	0.217	4.18*	1.505	1.01*	0.010	0.06	0.067
Single	Slope	191.7*	0.328	2.15	2.277	1.01*	0.010	0.06	0.067
Outliers	Intercept & Slope	191.9*	0.187	4.21*	1.301	1.01*	0.010	0.07	0.068

Table 3 shows the regression coefficients estimates for level II, equations 2 and 3. The standard error estimates of all parameters increased relative to the dataset with no outliers. The outliers had little effect on both γ_{11} and γ_{10} and did not alter the statistical test results. This indicates that the outliers generated under these conditions had little effect on the relationship between X and Y when W=0, γ_{10}, or the mean difference in the X-Y slopes across levels of W, γ_{11}. However, the outliers did have an effect on γ_{01}, the mean difference in Y across levels of W. Both the values for γ_{01} and its standard error changed such that in all datasets with 10% outliers the statistical test of γ_{01} was no longer significant. The net effect of the outliers then, was making the tests of γ_{01} more conservative. For the single outlier datasets, only the outlying slope changed the results of the significance tests of γ_{01}.

Discussion

When a cluster of outlying level-II observations are added to a multilevel analysis, the effects on the estimation of level II fixed effects can be dramatic. In school-effectiveness research, this would be manifested if a cluster of schools had extreme means on Y, aberrant relationships between X and Y in a cluster of schools, or if a cluster of schools had both extreme means and extreme X-Y relationships. Under the conditions investigated in this study the changes in the parameter estimates and their standard errors resulted in conservative tests of significance. Adding a single outlying school naturally had less of an effect, however, results of tests of significance were still changed in the presence of a single outlying slope. Further, it is conceivable that similar results could arise for single outliers of the intercept and the slope and intercept for more extreme outliers.

Future Research

This study suggests a more extensive study to ascertain how outliers of differing magnitudes affect parameter estimation in multilevel analysis is needed. It would also be of interest to study the effects of other types of outliers, such as leverage point outliers, on parameter estimation in multilevel analysis.

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Footnotes

[1]Reliability in HLM refers to the ratio of parameter variance to total variance.

$$\text{Reliability } (\hat{\beta}_q = \frac{1}{J}\sum_{j=1}^{J} \tau_{qq} / (\tau_{qq} + v_{qq}) \qquad (4)$$

for each q=0,...,Q (Bryk & Raudenbush, 1992, p.43) and where t_{qq} = parameter variance and v_{qq} = error variance (Bryk & Raudenbush, p.34).